

AN INVESTIGATION OF THE VIRTUAL MASS
OF A CYLINDER VIBRATING IN WATER

DAVID A. ROGERS
AND
MACLEAN C. SHAKSHOBER
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An Investigation of the Virtual Mass

of a

Cylinder Vibrating in Water

by

DAVID A. ROGERS

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LIEUTENANT, U. S. NAVY

B. S. UNITED STATES NAVAL ACADEMY (1945)

and

MACLEAN C. SHAKSHOBER

LIEUTENANT, U. S. Navy

Submitted to the Department of Naval Architecture and Marine Engineering on
May 25, 1953 in partial fulfillment of the requirements for the degree of
Naval Engineer.

Professor F. M. Lewis,---Thesis Supervisor

Chairman of Department Committee on Graduate Students

ABSTRACT

AN INVESTIGATION OF THE VIRTUAL MASS OF A CYLINDER
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DAVID A. ROGERS

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LIEUTENANT, U.S. NAVY

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Submitted to the Department of Naval Architecture and Marine Engineering on May 25, 1953 in partial fulfillment of the requirements for the degree of Naval Engineer.

The object of this thesis is to experimentally investigate the virtual mass of a hollow cylinder vibrating in water.

A lucite cylinder was magnetically vibrated in air and water at various length to diameter ratios and the frequency of vibration for as many modes as possible, up to five, recorded. No attempt was made to measure amplitude. The ratio of added water mass to displaced water mass was computed from the frequencies and compared with analytical results.

The investigation shows that end effects have a very great influence on virtual mass. As the length to diameter ratio is decreased, the added virtual water mass is decreased. There is also a decrease in virtual water mass as frequency is increased at constant length to diameter ratios.

A ratio of the measured virtual water mass to analytical, called K, was computed and found to be a function of L/D and mode number.

It is recommended that further investigations using bodies of revolution whose ends have zero area such as ellipsoids be made. It would be desirable to use equipment to permit measuring amplitude as well as frequency.

Where data is available, it is recommended that an attempt to calculate the frequencies of an actual hull such as a submarine be made, correcting the analytical virtual water mass by the applicable K values.

Thesis Supervisor: Professor F. M. Lewis
Title: Professor of Marine Engineering

AN INVESTIGATION OF THE EFFECTS OF A CHANGING
VIBRATORY FIELD

by

WILLIAM V. BRIDGES

DAVID L. BRIDGES

LESTER W. BRIDGES

LESTER W. BRIDGES

Submitted to the Department of Physics (Acoustics and Vibration Engineering) on
May 22, 1953 in partial fulfillment of the requirements for the degree of
Master of Science.

The object of this thesis is to experimentally investigate the effects
of a harmonic vibrator vibrating in water.

A simple vibrator was experimentally vibrated in air and water at various
frequencies to determine the effects of the frequency of vibration on the sound
pressure level, up to 1000 cycles per second. It was found that the sound
pressure level in water was higher than in air at the same frequency.

The investigation shows that the effects of a very small influence
on a system, as the case of the vibrator, is to increase the sound
pressure level in water. There is also a decrease in the sound
pressure level in air at the same frequency.

A study of the sound pressure level in water was also made. It was
found that the sound pressure level in water was higher than in air at the same
frequency.

It is recommended that further investigation be made on the effects of
vibration on the sound pressure level in water. It would be desirable
to see whether the sound pressure level in water is affected by the frequency
of vibration.

These data are useful, it is recommended that an attempt be made to
determine the effects of a vibrator on the sound pressure level in water. The
effects of a vibrator on the sound pressure level in water are of interest
to the acoustics engineer.

These data are useful, it is recommended that an attempt be made to
determine the effects of a vibrator on the sound pressure level in water.

Cambridge, Massachusetts
May 25, 1953

Professor Earl B. Millard
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the degree of Naval Engineer,
we herewith submit a thesis entitled "An Investigation of the Virtual Mass
of a Cylinder Vibrating in Water."

Respectfully,

David A. Rogers
Lieutenant, U. S. Navy

MacLean C. Shakshober
Lieutenant, U. S. Navy

University of California
Berkeley, California
May 22, 1952

Professor Earl R. Riefler
University of the Pacific
Stockton, California

Dear Sir:

In accordance with the requirements for the degree of Master of Arts,
we request that you submit a thesis entitled "An Investigation of the History of
of a California Community in Mexico."

Respectfully,

John A. Riefler
Chairman, M. A. Program

Thomas W. Riefler
Chairman, M. A. Program

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I. INTRODUCTION

When immersed in a dense fluid, such as water, a body vibrates as though it had undergone an increase in mass. This increase in mass is due to the flow of fluid about the body as it moves.

Several analytical methods of computing the virtual mass of a body vibrating in a dense fluid have been presented.^(1, 2) These methods are based on the assumption of potential flow about the body. Unless the body is of uniform shape with pointed ends, i.e. an ellipsoid, it is not presently possible to compute analytically the virtual mass because of the flow about the ends being highly rotational.

A submerged submarine must vibrate as a free-free body at a frequency determined by its virtual mass as described above. When vibrating horizontally, its virtual mass will be somewhat lower than that computed on the basis of no end losses because of flow about the ends.

Professor Frank M. Lewis has presented a method for determining the virtual mass based on an ellipsoid which may be corrected for other shapes.⁽¹⁾ Dr. H. M. Schauer of the Underwater Explosion Research Division, Norfolk Naval Shipyard, has done likewise for a cylinder with no end flow. Mr. E. B. Moullin and Mr. A. D. Browne, in a paper presented before the Cambridge Philosophical Society in 1928⁽³⁾ gave the results of their investigation of the periods of a free-free bar of rectangular cross section vibrating in water. In their experiments they used long bars which had length to depth ratios of from 26 to 39. Using such long bars they found that flow about the ends did not have any appreciable effect on the virtual mass as analytically computed. However, they did not investigate lower length to depth ratios. They found that the virtual mass is not affected by depth when below about two diameters.

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[illegible]

The following is a report on the experimental determination of the virtual mass of a circular cylinder for several length to diameter ratios while vibrating in water.

The following is a report on the experimental determination of the
 critical mass of a spherical cylinder for various lengths of cylinder with
 ends flat.

The following table gives the results of the experiments. The first column
 gives the length of the cylinder in inches. The second column gives the
 critical mass in grams. The third column gives the critical mass in
 grams per cubic centimeter. The fourth column gives the critical mass
 in grams per square centimeter. The fifth column gives the critical mass
 in grams per linear centimeter. The sixth column gives the critical mass
 in grams per unit area. The seventh column gives the critical mass
 in grams per unit volume. The eighth column gives the critical mass
 in grams per unit length. The ninth column gives the critical mass
 in grams per unit surface. The tenth column gives the critical mass
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 in grams per unit surface. The ninety-seventh column gives the critical mass
 in grams per unit volume. The ninety-eighth column gives the critical mass
 in grams per unit length. The ninety-ninth column gives the critical mass
 in grams per unit surface. The hundredth column gives the critical mass
 in grams per unit volume.

II. PROCEDURE

In this section the steps followed to obtain the desired results are described. The procedure consists of two parts, experimental and analytical. Details of the procedure are presented in the Appendix.

Experimental Procedure

From "The Theory of Sound" by Rayleigh⁽⁴⁾ the appropriate equations were used to obtain the nodal and anti-nodal points of a free-free bar. Using the frequency equation for a free-free bar

$$f = \frac{n^2}{2\pi} \left[\frac{EK^2}{\delta} \right]^{1/2} \quad (1)$$

the first five modes were computed to give an approximation of the natural frequencies.

To vibrate the cylinder mechanically would require a motor with a speed range of 1800 to 60,000 revolutions per minute. For this reason, magnetic vibration of the bar was by far the preferred method. Schematics of the apparatus are shown in Figure I.

A Lucite plastic tube 52.7 inches long with an outer diameter of 2 inches and an inner diameter of 1.75 inches was used for the first test. On the end of the cylinder was wrapped some small diameter soft iron wire to permit magnetic excitation. The amount of wire was not great enough to affect the frequency or mass of the bar. By experimenting with various types of pickups, it was found that a seismic crystal gave the best results. The pickup was very light in weight and very sensitive to vibration. This particular pickup was a Brush seismic crystal used on the sounding board of an electric guitar. The pickup was mounted on the inside of the cylinder at the opposite end of

Experimental Results

In this section the results of the experiments are described. The general character of the results is summarized in Table I. Details of the experiments are given in the Appendix.

General Results

From the results of the experiments it was found that the results were very similar to those obtained in the case of a single crystal. The results are summarized in Table I.

$$N \left[\frac{1}{2} \left(\frac{1}{2} \right) \right] \frac{1}{2} = 1$$

(1)

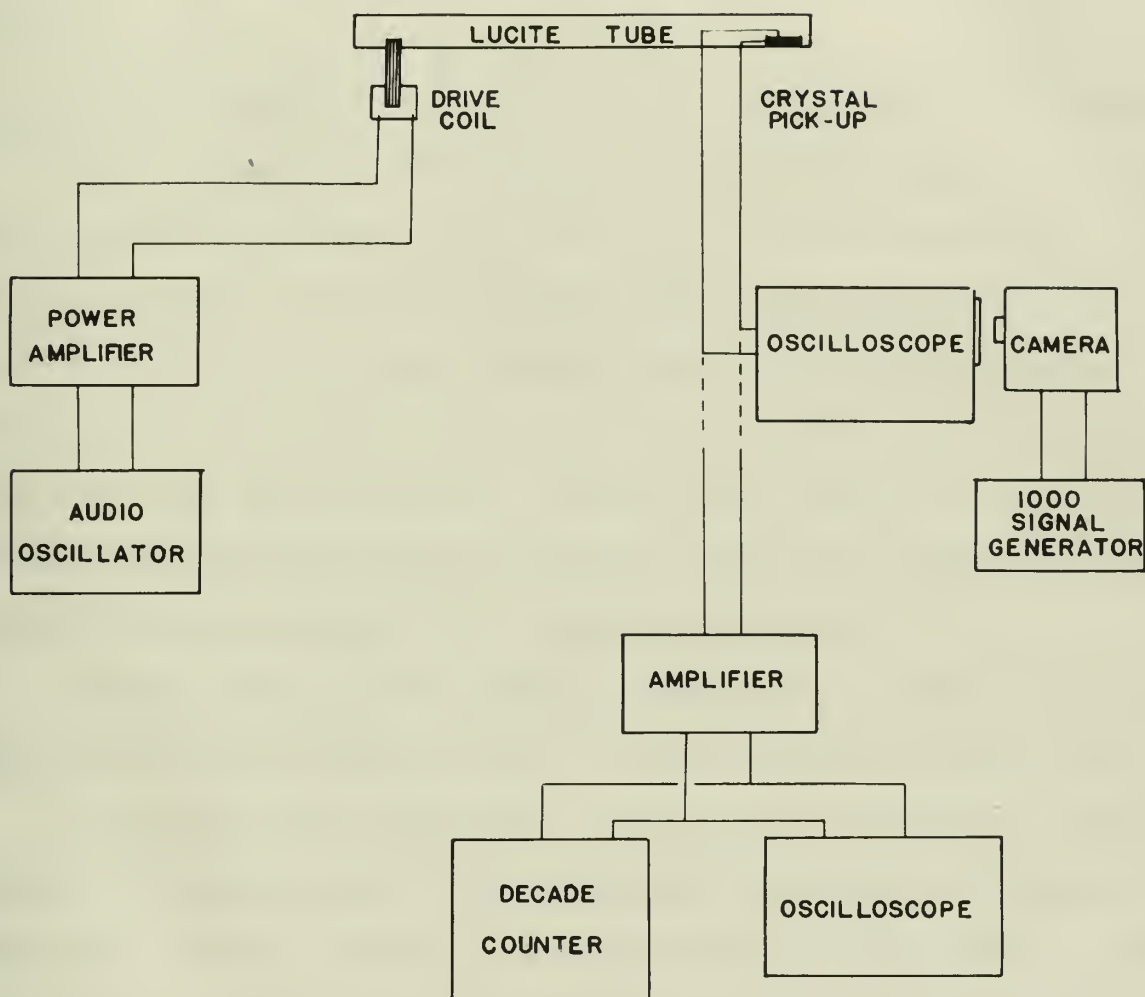
The first three results are given in Table I. The results are summarized in Table I.

To obtain the results summarized in Table I, the results were obtained from the experiments. The results are summarized in Table I.

The results are given in Table I. The results are summarized in Table I. The results are summarized in Table I.

FIG. I
SCHEMATIC

MAY 8, 1953, MCS



the exciting wire. A small hole was drilled at a nodal point through which the wires from the pickup were run.

An audio oscillator with a frequency range of 20 to 20,000 cycles per second was used to drive an electro-magnet. The output of the audio oscillator was amplified in a power amplifier and this was used to drive the electro-magnet. The output of the crystal pickup was put into a cathode ray oscilloscope where the signal was peaked for resonance. Unfortunately, due to radiation, frequencies above 1600 cycles per second could not be detected.

Two different types of frequency measurement were employed to compute the frequency of vibration, both of which gave very accurate and like results. The first method was to take the output of the pickup and put it on the vertical plates of the CRO. When a resonant signal was obtained, a picture of the frequency was taken with no horizontal sweep. The camera used was a very high speed model with no shutter. The camera had a built-in timing light of 1000 cycles per second which showed on the film. The frequency was then computed from the developed film by counting the cycles.

The other method of determining the frequency was to take the amplified output of the crystal pickup and put it into an electronic decade counter.

To compute the air frequencies, the cylinder was suspended by strings located at the nodal points. The electro-magnet was placed as close as possible to the soft iron windings. To prevent banging of the cylinder, a rubber band was used as a standoff. The frequencies were recorded as described above.

For the water tests, the cylinder was immersed seven diameters in the towing tank in the M. I. T. Hydrodynamics Laboratory. This depth ensured that no surface effects would be present. The cylinder was anchored by two strings at the nodes.

the working fluid. A small hole was drilled at a radial point between which the stress from the bridge was zero.

An audio oscillator of 1000 cycles per second was used to drive an electro-magnet. The output of the audio oscillator was amplified in a power amplifier and this was used to drive the electro-magnet. The output of the crystal pickup was fed into a vacuum tube oscillator where the signal was needed for transmission. Inductively coupled to this oscillator, a second 1000 cycle per second coil was fed back to the oscillator. The different types of frequency measurement were employed to compare the frequency of vibration with of which give very accurate and low results. The first method was to take the output of the pickup and put it in the center of the glass of the SGO. When a resonant signal was obtained, a picture of the frequency was taken with an oscilloscope. The second used was a very high speed wheel with an indicator. The wheel was a white disk with light of 1000 cycles per second which showed in the film. The frequency was then compared from the developed film by counting the cycles.

The other method of determining the frequency was to take the amplified output of the crystal pickup and put it into an electronic bridge circuit. To compare the air frequencies, the oscillator was connected by bridge located at the radial bridge. The electro-magnet was placed as shown in figure 1 to the left from the bridge. To prevent loading of the oscillator, a rubber band was used as a standard. The frequencies were recorded as described above.

For the water tests, the cylinder was mounted with the piston in the testing tank in the M. I. T. Hydraulics Laboratory. This setup was used for no further effects could be possible. The cylinder was actuated by two springs at the bottom.

The same procedure was followed using cylinders of 40, 38, 30, 28, and 22 inches, and for a 42-inch bar with six-inch conical ends.

Analytical Procedure

The ratio of the added water mass to displaced water mass, M_v/M_o , was computed directly from the observed frequencies as explained in Details of Procedure.

A correction factor K was then calculated, where K is the ratio of measured M_v/M_o to that computed by Dr. H. M. Schauer. (2)

CHAPTER III

The results are plotted on Figure 11 through 14.

The following are plotted on Figure 11 through 14.

Figure 11: Plot of $\log \frac{1}{1 - \alpha}$ versus $\log \frac{1}{1 - \beta}$. The data points show a linear relationship with a slope of approximately 1.0.

Figure 12: Plot of $\log \frac{1}{1 - \alpha}$ versus $\log \frac{1}{1 - \beta}$. The data points show a linear relationship with a slope of approximately 1.0.

Figure 13: Plot of $\log \frac{1}{1 - \alpha}$ versus $\log \frac{1}{1 - \beta}$. The data points show a linear relationship with a slope of approximately 1.0.

Figure 14: Plot of $\log \frac{1}{1 - \alpha}$ versus $\log \frac{1}{1 - \beta}$. The data points show a linear relationship with a slope of approximately 1.0.

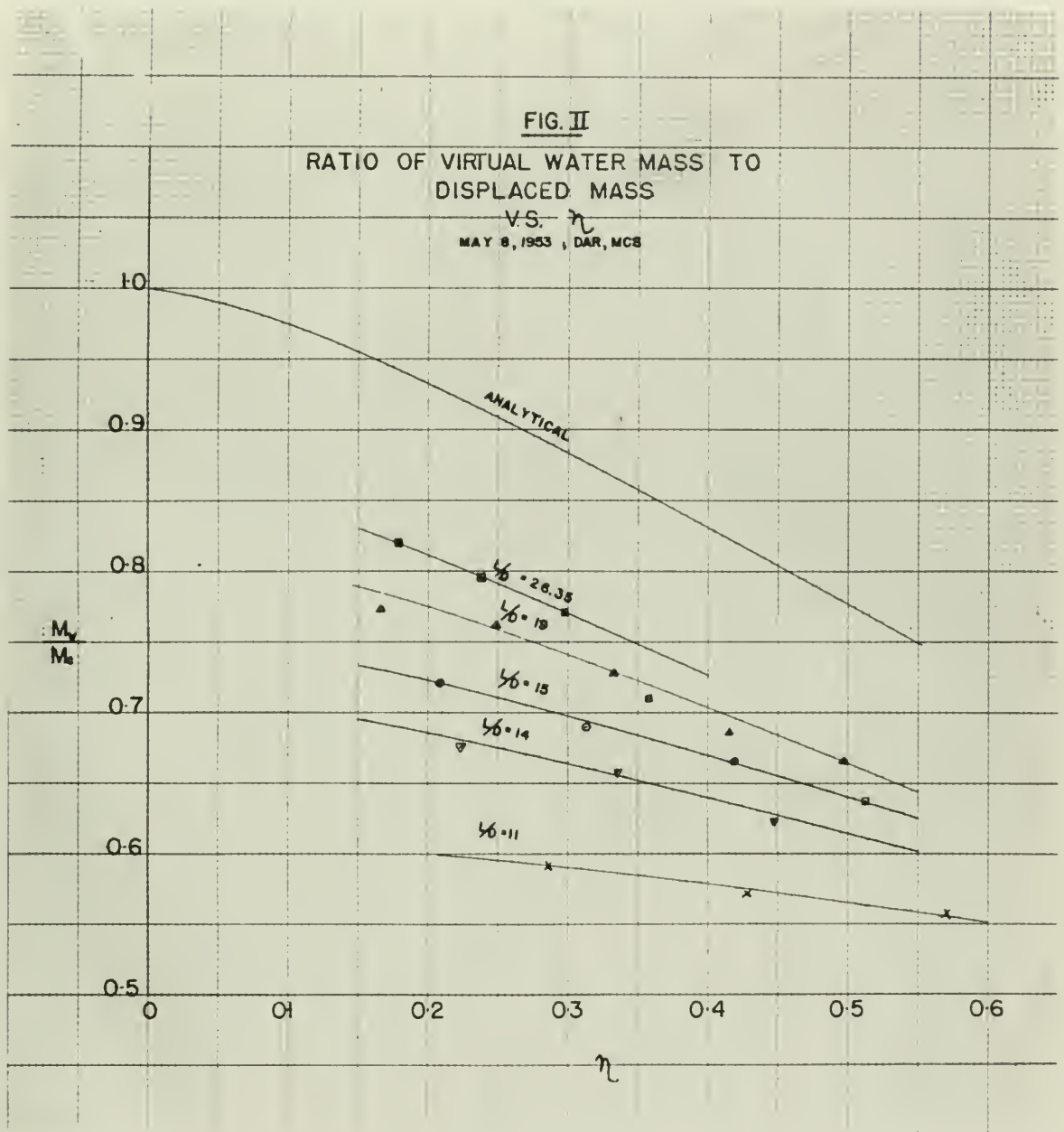


FIG. III
MASS RATIO V.S. L/D FOR
CONSTANT η
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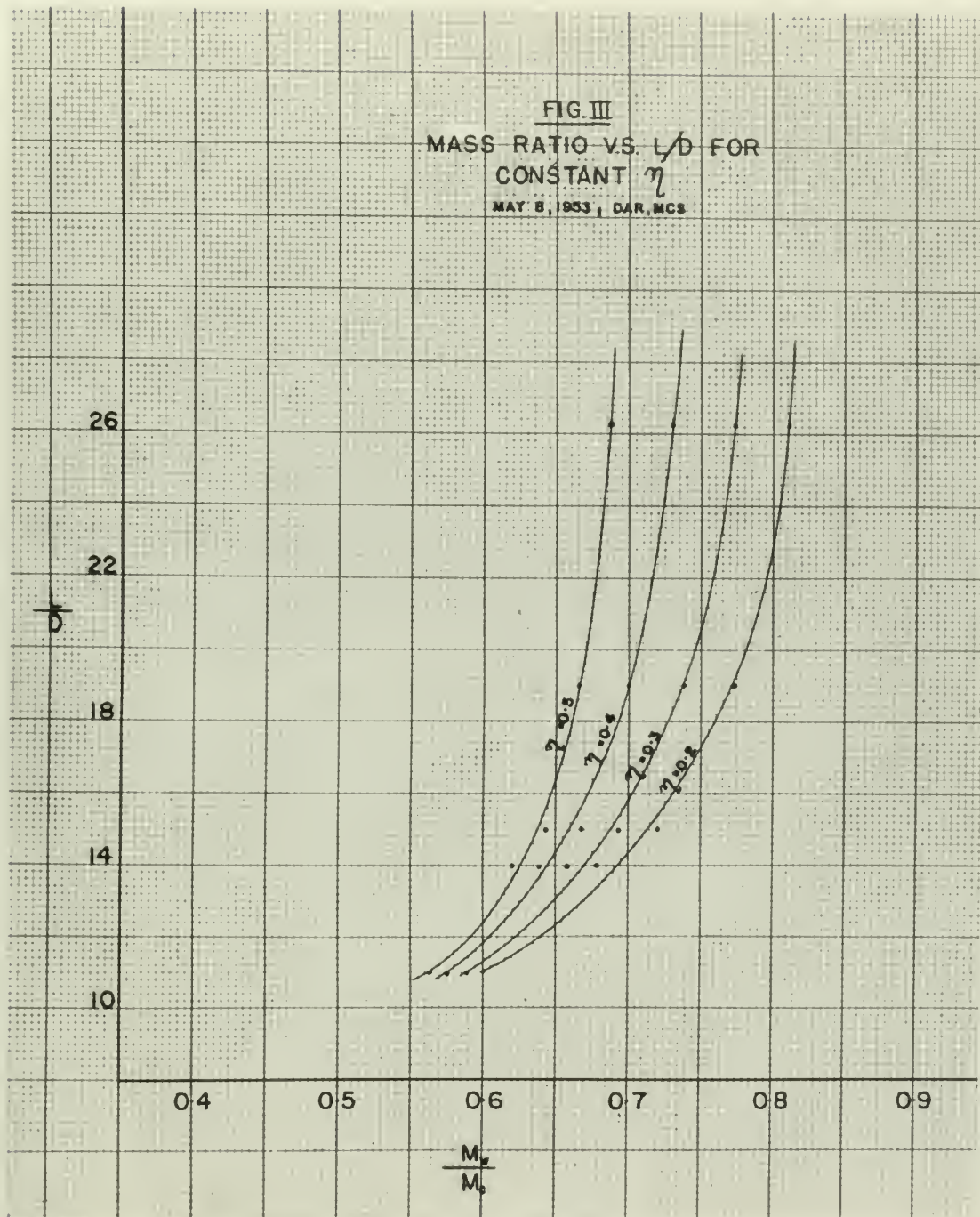


FIG. IV

K V.S. η FOR CONSTANT $\frac{L}{D}$
MAY 8, 1953, DAR, MGS

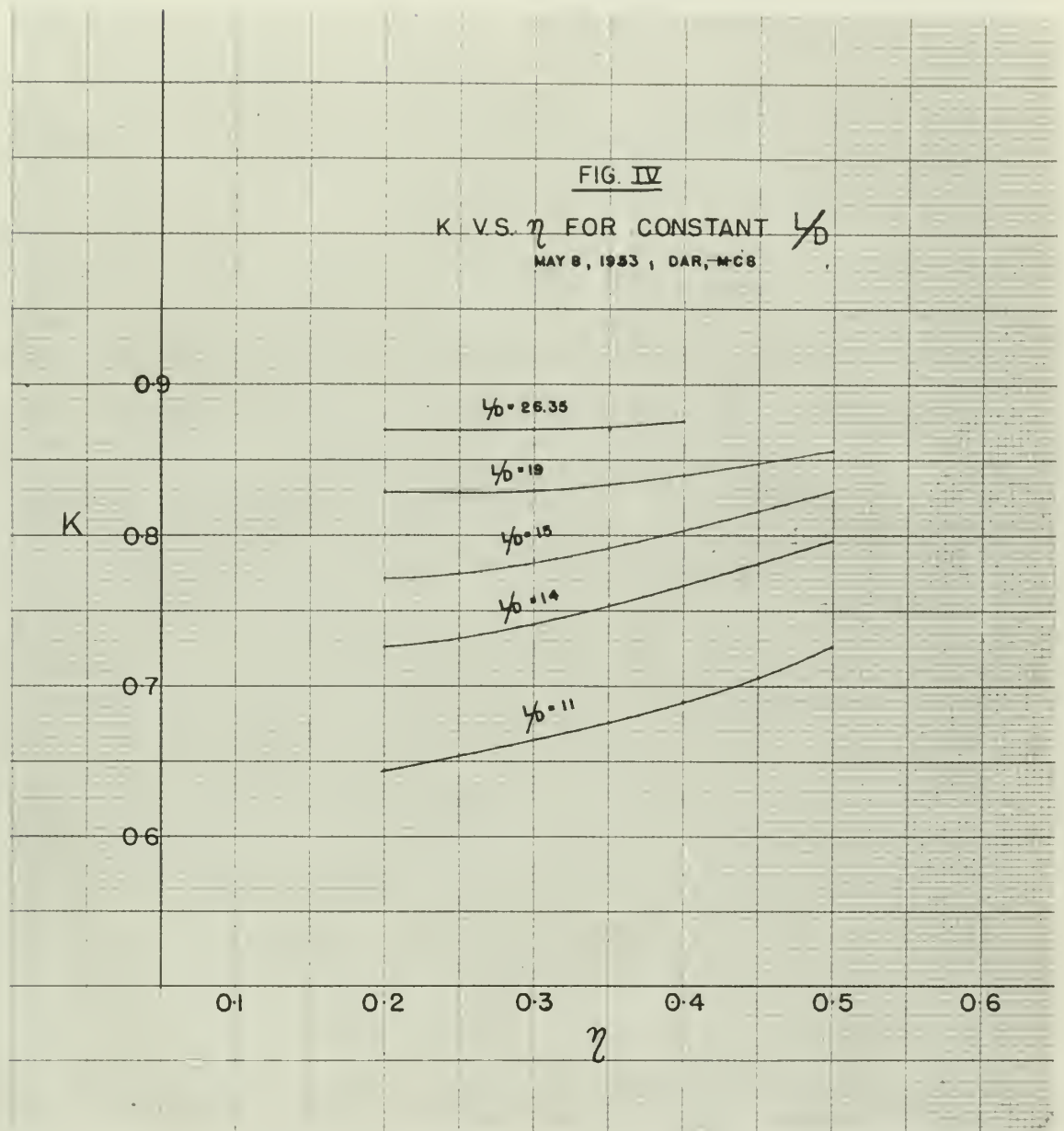
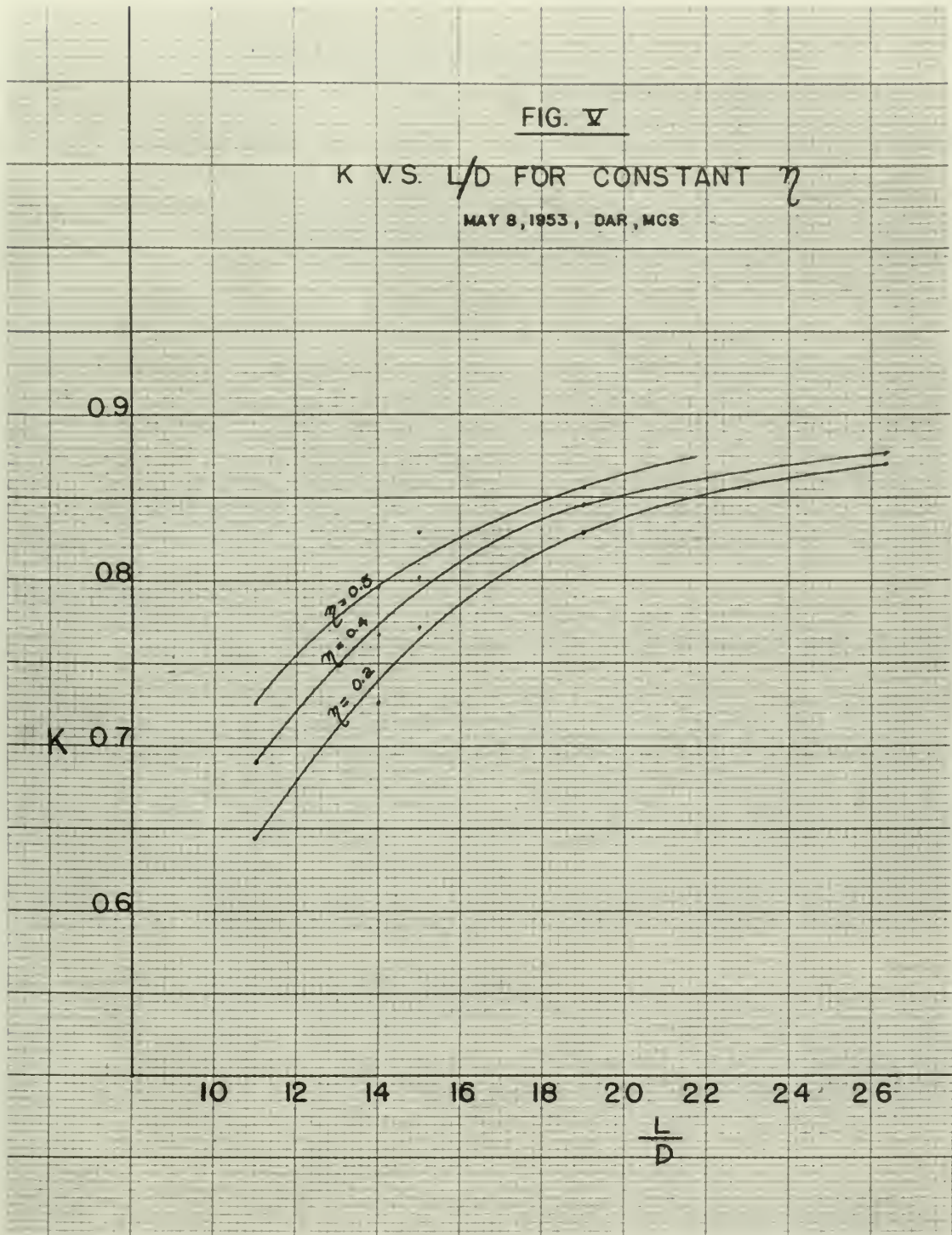


FIG. V

K V.S. L/D FOR CONSTANT η

MAY 8, 1953, DAR, MCS



IV. DISCUSSION OF RESULTS

The results shown plotted in figures II through V indicate that the ends have a large influence on the value of the ratio of added water mass to the displaced water mass, i.e. M_w/M_c . As the length to diameter ratio is decreased, the value of K, (the ratio of measured M_w/M_c to the M_w/M_c calculated by Dr. Schauer), decreases rapidly for a given value of η . However, K increases for increasing η at constant L/D.

K is based on an analytical curve which assumes a potential flow, and therefore irrotational, with no flow about the ends. Thus K may be considered a measure of the end flow and of rotationality. For a cylinder with flat ends a large portion of K can probably be attributed to flow of the fluid about the ends.

Just how much of the difference from analytical values can be attributed to end flow is difficult to say from these experiments. An inspection of the test results of the cylinder with conical ends indicates that end flow is not the only contributing factor since the area at the ends of the cones is substantially zero. However, due to the abrupt change in shape of the body where the cones are joined to the cylinder, a certain amount of spilling of fluid toward the ends will occur.

Due to the large effects of L/D on the value of M_w/M_c it appears that a direct application of Dr. Schauer's equation is not feasible unless the body is very long relative to its breadth.

Professor F. M. Lewis⁽¹⁾ has shown that the added water mass per foot of length for an ellipsoid is

IV. DISCUSSION OF RESULTS

The results shown in Table II show that the
 only large difference in the value of \sqrt{V} is
 to the highest value, i.e. $\sqrt{V} = 0.5$. As the length of the
 distance, the value of \sqrt{V} (the value of \sqrt{V} is the \sqrt{V} value
 related to the distance), decreases rapidly for a given value of \sqrt{V} .
 It is based on an empirical curve which shows a constant \sqrt{V} and
 therefore constant, with no other than the value. Thus it can be seen
 that a measure of the rate of change of \sqrt{V} for a given value
 that with a large portion of it was probably in relation to the
 field about the body.

Just how much of the difference from empirical values can be attrib-
 uted to the fact is difficult to say from these experiments. An inspection
 of the last results of the relation with the value of \sqrt{V} indicates that
 this is not the only contributing factor since the value of \sqrt{V} at the
 corner is substantially zero. However, due to the change in shape of
 the body where the corner was placed in the cylinder, a certain amount of
 splitting of field lines would have been caused.

Due to the large values of \sqrt{V} at the value of $\sqrt{V} = 0.5$ it appears that
 a direct application of the Gaussian's equation is not feasible unless the
 body is very long relative to its breadth.

Professor F. R. Jones⁽¹⁾ has shown that the value of \sqrt{V} for
 that of length for an ellipsoid is

$$M_w = CJ\pi B^2 \gamma_w \quad (2)$$

where

C = Section inertia coefficient

B = Half beam or radius of ellipsoid at section

γ_w = Specific weight of fluid

J = $\frac{\text{Actual K. E. surrounding fluid}}{\text{K. E. of fluid if flow is two dimensional}}$

Now for a cylinder C = 1 and we can write

$$M_w = J\pi R^2 \gamma_w L \quad (3)$$

and

$$\frac{M_w}{M_c} = \frac{J\pi R^2 \gamma_w L}{\pi R^2 \gamma_w L} \quad (4)$$

so that

$$J = \frac{M_w}{M_c} \quad (5)$$

It was found that if Prof. Lewis' J factor is multiplied by our K, a fairly decent approximation to the actual frequency of a submerged body can be computed. A computation was made of the submerged frequency of a 42-inch body having 6-inch cones at each end of a cylindrical middle body 30 inches long.

Using Prof. Lewis' J factor multiplied by K for an L/D ratio of 21 at $\eta = .15$, the computed two noded frequency was 68.9 c.p.s. and the experimentally measured value was 70 c.p.s. For the three noded frequency the computed value was 187 c.p.s. and the measured value was 188 c.p.s.

Although this method gives very good results in the case tested, further investigation is required to determine its limits of application. Similar results would have been found using Dr. Schauer's M_w/M_c in a like manner.

(2)

$$N = \frac{1}{2} \frac{d^2 \epsilon}{d\lambda^2}$$

where ϵ = dielectric constant

λ = half wave or twice of thickness of section

$\frac{d^2 \epsilon}{d\lambda^2}$ = second derivative of ϵ with

respect to λ . ϵ is the dielectric constant of the material.

For a dielectric $\epsilon = 1$ and we can write

(3)

$$N = \frac{1}{2} \frac{d^2 \epsilon}{d\lambda^2}$$

and

(4)

$$\frac{N}{\lambda} = \frac{1}{2} \frac{d^2 \epsilon}{d\lambda^2} \frac{1}{\lambda}$$

(5)

$$\frac{N}{\lambda} = \frac{1}{2} \frac{d^2 \epsilon}{d\lambda^2}$$

so that

It was found that if ϵ is not a function of λ , the value of N is zero. This is because the derivative of a constant is zero. A constant value of ϵ is obtained by having a uniform material. If ϵ is a function of λ , the value of N is not zero. This is because the derivative of a function is not zero. The value of N is determined by the derivative of ϵ with respect to λ .

Using the values of ϵ for the different materials, the values of N were calculated. The values of N were found to be very small. This is because the derivative of ϵ with respect to λ is very small. The values of N were found to be very small for all the materials.

Although this method gives very good results in the case of dielectric materials, it is not applicable to all materials. This is because the derivative of ϵ with respect to λ is not zero for all materials. The results would have been better if the derivative of ϵ with respect to λ was not zero.

V. RECOMMENDATIONS

In view of the large discrepancy between values of theoretical mass found by analytical means and those measured, it is recommended that an investigation similar to this be made of families of bodies of revolutions having ends whose section decreases to zero.

The measurement of the virtual mass of bodies which do not have complete radial symmetry is much more difficult since pains must be taken to insure that the vibrations are limited to the plane desired. However, it is recommended that where data exists on the vibration of submerged bodies such as submarines, an attempt be made to apply the correction, K , to compute their frequencies.

VI. APPENDIX

CHAPTER IV

The first part of the chapter is devoted to a discussion of the

general principles of the theory of the function of the

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CHAPTER IV

A. DETAILS OF PROCEDURE

Selection of Tubing

Since the object of this investigation is to determine the virtual added mass of water, it is desirable that the cylinder be made of a material having a low specific gravity. Thus the added water mass will be a large fraction of the total virtual mass of the body. In order that the frequencies of the cylinder will not be excessively high, the material should have a relatively low modulus of elasticity.

As a result of these considerations, it was decided that the cylinder should be constructed of Lucite. The specific gravity of Lucite is 1.18 and has a modulus of elasticity in the order of 5×10^5 pounds per square inch.

Effect of Added Masses

The weight of the soft iron wire was .373 ounces and covered three-fourths of an inch of the cylinder. The weight of the crystal pick-up and clamp used to hold it securely in place was .303 ounces. The weight of the plastic cylinder was .497 ounces per inch. The effect of these added masses on the frequencies was assumed negligible.

Comparison of Frequency Measurement

The high speed movie camera method of frequency measurement proved very satisfactory, but also required a great deal of time. The camera had a neon bulb timing light built in which was energized by a 1000 cycle audio oscillator. The output of the oscillator was amplified through a CRO which provided a means of adjusting the light to the desired brightness. The only errors

[illegible][illegible]

The weight of the soil was also measured and found to be 100 lbs. The weight of the soil was also measured and found to be 100 lbs.

Department of Psychology

[illegible]

that could be made in this method were (a) errors in counting the film and (b) frequency drift in the oscillator. Two separate runs were made on each length tested.

In order to speed up the experiment and reduce the amount of labor involved, it was decided to try frequency measurement with an electronic decade counter. If there is any frequency drift of the audio oscillator, the results of the decade counter will be more accurate than the movie camera since the decade counter records every second noting any change which may occur.

Boundary Effects in Water

To check for wall effects, the cylinder was submerged in the stability tank and the test rerun. The results obtained were the same as those observed in the towing tank test. A further check was made by varying the distance of the cylinder from the walls in the stability tank. Again no difference was noted. No attempt was made to vibrate the cylinder within four diameters of the wall.

Comparison with Different Cylinder

The necessity of re-checking the 30-inch cylinder arose after it had been cut down to 22 inches. A new 30-inch plastic tube was re-wound with approximately the same amount of wire as before. Both the air and water tests agreed identically with the previous 30-inch test.

Damping Effects of Rubber Band Stand-off

To ascertain whether the rubber band had any appreciable damping effect, the force necessary to stretch the rubber band one inch and the force required to deflect the tube one inch were calculated. To stretch the rubber band

(b) Government will be the beneficiary. For example, some have said we must

It is not to be expected that the Government will be able to do more than to make a few suggestions, and to leave the matter to the discretion of the local authorities. It is not to be expected that the Government will be able to do more than to make a few suggestions, and to leave the matter to the discretion of the local authorities.

It should be well known, the opinion you expressed in the evidence
that the last year. The people of the world are now in a
position in the world. A further check was made by testing the
balance of the system from the world in the existing state. Again in
the system was found. It always was and is always the same thing.

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1. The University of Wisconsin has been authorized to issue a license to the University of Wisconsin Press to publish and distribute the University of Wisconsin Press Series in the History of the State of Wisconsin.

10. The results of the study show that the proposed method is effective in detecting and removing malicious code from the system. The results also show that the proposed method is able to detect and remove malicious code from the system in a timely manner.

one inch required a force of .61 pounds, and to deflect the cylinder one inch, a force of 454 pounds was necessary. The damping effect of the rubber band may be neglected.

Amplitude Measurement

The equipment used was not capable of measuring amplitudes. This is partly because the natural resonant period of the pick-up, 980 cycles per second, was within the range of the frequencies measured. The primary difficulty was that it was not possible to maintain a constant exciting force on the cylinder. There also was no means of accurately measuring the exciting force. A constant exciting force was not possible because a higher force was required to excite the higher modes, but this same force would cause the cylinder to be drawn hard against the faces of the electro-magnet poles at the lower modes. It was necessary therefore to start with relatively low driving forces at low modes and increase the force to excite the higher frequencies.

Computation of Virtual Mass

The virtual mass was computed assuming negligible damping so that

$$\frac{M_V}{M_T} = \left(\frac{f_1}{f_2} \right)^2 \quad (6)$$

where

M_V = Virtual Mass

M_T = Mass of Tube

f_1 = Frequency in Air

f_2 = Frequency in Water

Let

M_W = Added Water Mass

M_C = Displaced Water Mass

then

$$M_V = M_W + M_T \quad (7)$$

$$M_W = M_T \left[\left(\frac{f_1}{f_2} \right)^2 - 1 \right] \quad (8)$$

and

$$\frac{M_W}{M_C} = \frac{M_T}{M_C} \left[\left(\frac{f_1}{f_2} \right)^2 - 1 \right] \quad (9)$$

Thus the ratio $\left(\frac{M_W}{M_C} \right)$ may be computed from the measured values of the frequencies.

Computation of K

Dr. H. M. Schauer⁽²⁾ has derived an analytical expression for the mass ratio, $\frac{M_W}{M_C}$, as follows:

$$\frac{M_W}{M_C} = \frac{1}{1 + \eta \frac{iH_0(i\eta)}{-H_1(i\eta)}} \quad (10)$$

where H_0 and H_1 are Hankel functions and other symbols as previously defined.

The variation of the measured mass ratio from the above will be a function of η and the $\frac{L}{D}$ ratio. This ratio can be expressed as a ratio, K

$$K = \frac{\text{measured mass ratio}}{\text{analytical mass ratio}} = \frac{(M_W/M_C)_M}{(M_W/M_C)_A} \quad (11)$$

and the other side of the equation is the same as the left side.

(7)

(8)

$$\left[\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right] \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

(9)

$$\left[\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right] \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Thus the ratio $\frac{M}{H}$ is the same as the ratio of the

moments.

Definition of A

Dr. J. W. L. has defined an arbitrary constant for the case

ratio $\frac{M}{H}$ is defined as follows:

(10)

$$\frac{1}{A} = \frac{1}{\frac{M}{H}}$$

where H and M are the horizontal and vertical moments respectively.

The ratio of the moments is the same as the ratio of the

time of $\frac{1}{A}$ and the $\frac{1}{H}$ ratio. This ratio can be written as a ratio $\frac{1}{A}$.

and the other side of the equation is the same as the left side.

(11)

$$\frac{1}{A} = \frac{1}{\frac{M}{H}}$$

B. SUMMARY OF DATA AND CALCULATIONS

1. Definition of Symbols.

f_1 = Observed air frequency, cycles per second

f_2 = Observed water frequency, cycles per second

M_w = Added mass of water

M_o = Mass of displaced water

L = Length of cylinder, inches

D = Diameter of cylinder, inches

$\eta = \frac{D}{2L} (m + 1)$

m = Mode number

SECTION 101.101 TO 101.105

1. Section 101.101 to 101.105

2. Section 101.101 to 101.105

3. Section 101.101 to 101.105

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17. Section 101.101 to 101.105

18. Section 101.101 to 101.105

19.

20. Section 101.101 to 101.105

21.

2. Calculated values of $\frac{M_v}{M_c}$.

MODE	f_1	f_2	$\frac{M_v}{M_c}$	η
1	61.7			
2	170.5	86.2	0.82	0.179
3	332.0	169.1	0.795	0.238
4	539.5	279.0	0.770	0.298
5	787.5	419.0	0.709	0.358

TABLE I

Test Results 52.7-inch Cylinder $L/D = 26.35$

MODE	f_1	f_2	$\frac{M_v}{M_c}$	η
1	104.5	54	.792	.157
2	290.5	151	.768	.236
3	562	295	.742	.314
4	893	476	.711	.392
5	1277	692	.682	.471

TABLE II

Test Results of 40-inch cylinder $L/D = 20$

θ	$\frac{1}{\theta}$	$\frac{1}{\theta^2}$	$\frac{1}{\theta^3}$	RTN
100.0	0.0100	0.0001	0.00001	1
90.0	0.0111	0.000123	0.0000137	2
80.0	0.0125	0.000156	0.0000196	3
70.0	0.0143	0.000204	0.0000282	4
60.0	0.0167	0.000278	0.0000393	5

TABLE I

Test results of 10-sec. test - P. 12

θ	$\frac{1}{\theta}$	$\frac{1}{\theta^2}$	$\frac{1}{\theta^3}$	RTN
100.0	0.0100	0.0001	0.00001	1
90.0	0.0111	0.000123	0.0000137	2
80.0	0.0125	0.000156	0.0000196	3
70.0	0.0143	0.000204	0.0000282	4
60.0	0.0167	0.000278	0.0000393	5

TABLE II

Test results of 10-sec. test - P. 12

MODE	f_1	f_2	$\frac{M}{M_0}$	η
1	114	59	0.773	0.166
2	313	163	0.761	0.249
3	602	319	0.727	0.332
4	943	509	0.685	0.415
5	1351	739.5	0.665	0.497

TABLE III

Test Results 38-inch Cylinder $L/D = 19$

MODE	f_1	f_2	$\frac{M}{M_0}$	η
1	183	97.5	0.720	0.209
2	496.5	268.5	0.689	0.314
3	936	512	0.665	0.419
4	1454	808	0.637	0.523

Table IV

Test Results 30-inch Cylinder $L/D = 15$

MODE	f_1	f_2	$\frac{M}{M_0}$	η
1	205	112	0.674	0.224
2	553	305	0.657	0.336
3	1036	581.5	0.622	0.448

TABLE V

Test Results 28-inch Cylinder $L/D = 14$

MODE	f_1	f_2	$\frac{M}{M_0}$	η
1	299	172	0.590	0.286
2	800	465	0.570	0.428
3	1457	855	0.556	0.571

TABLE VI

Test Results 22-inch Cylinder $L/D = 11$

μ	$\frac{\mu}{\sigma}$	ϵ_1	ϵ_2	Order
100.0	100.0	100	100	1
99.0	99.0	100	99	2
98.0	98.0	100	98	3

TABLE I

Test results for $\mu = 100$ and $\sigma = 1$

μ	$\frac{\mu}{\sigma}$	ϵ_1	ϵ_2	Order
100.0	100.0	100	100	1
99.0	99.0	100	99	2
98.0	98.0	100	98	3

TABLE II

Test results for $\mu = 100$ and $\sigma = 1$

η	$1H_o(1\eta)$	$-H_1(1\eta)$	$\frac{1H_o(1\eta)}{-H_1(1\eta)}$	$\eta \frac{1H_o(1\eta)}{-H_1(1\eta)}$	$1+\eta \frac{1H_o(1\eta)}{-H_1(1\eta)}$	$\frac{M_w}{M_o}$
.120	1.431	5.20	.275	.0330	1.0330	.968
.131	1.376	4.75	.290	.0380	1.0380	.963
.175	1.198	3.50	.342	.0599	1.0599	.943
.196	1.128	3.11	.363	.0711	1.0711	.934
.261	.9956	2.275	.438	.114	1.114	.898
.262	.9954	2.265	.439	.115	1.115	.897
.327	.8238	1.763	.437	.153	1.153	.867
.349	.7865	1.635	.481	.168	1.168	.856
.392	.7209	1.425	.506	.198	1.198	.835
.437	.6609	1.249	.529	.231	1.231	.812
.524	.5639	.9928	.568	.298	1.298	.770

TABLE VIII

Analytical Calculation of Virtual Mass

By Dr. H. M. Schauer⁽²⁾

$$\frac{M_w}{M_o} = \frac{1}{1 + \eta \frac{1H_o(1\eta)}{-H_1(1\eta)}}$$

M_w = Added Water Mass

M_o = Mass of Displaced Water

M_T = Mass of Cylinder

$$\eta = (\pi+1) \frac{a}{L}$$

m = Mode Number

a = Radius

L = Length

p	$\frac{R}{S}$	$\frac{1}{S}$	$\frac{1}{R}$	ITEM
100.0	100.0	0.01	0.01	1
200.0	200.0	0.005	0.005	2
300.0	300.0	0.0033	0.0033	3

TABLE 1

Test results for the first three items

p	$\frac{R}{S}$	$\frac{1}{S}$	$\frac{1}{R}$	ITEM
100.0	100.0	0.01	0.01	1
200.0	200.0	0.005	0.005	2
300.0	300.0	0.0033	0.0033	3

TABLE 2

Test results for the first three items

η	$1H_o(1\eta)$	$-H_1(1\eta)$	$\frac{1H_o(1\eta)}{-H_1(1\eta)}$	$\eta \frac{1H_o(1\eta)}{-H_1(1\eta)}$	$1+\eta \frac{1H_o(1\eta)}{-H_1(1\eta)}$	$\frac{M_w}{M_o}$
.120	1.431	5.20	.275	.0330	1.0330	.968
.131	1.376	4.75	.290	.0380	1.0380	.963
.175	1.198	3.50	.342	.0599	1.0599	.943
.196	1.128	3.11	.363	.0711	1.0711	.934
.261	.9956	2.275	.438	.114	1.114	.898
.262	.9954	2.265	.439	.115	1.115	.897
.327	.8238	1.763	.437	.153	1.153	.867
.349	.7865	1.635	.481	.168	1.168	.856
.392	.7209	1.425	.506	.198	1.198	.835
.437	.6609	1.249	.529	.231	1.231	.812
.524	.5639	.9928	.568	.298	1.298	.770

TABLE VIII

Analytical Calculation of Virtual Mass

By Dr. H. M. Schauer⁽²⁾

$$\frac{M_w}{M_o} = \frac{1}{1 + \eta \frac{1H_o(1\eta)}{-H_1(1\eta)}}$$

M_w = Added Water Mass

M_o = Mass of Displaced Water

M_T = Mass of Cylinder

$$\eta = (\pi+1) \frac{a}{L}$$

n = Mode Number

a = Radius

L = Length

$\frac{H}{L}$	$\frac{f_1(1)}{f_2(1)} \frac{H}{L}$	$\frac{f_1(1)}{f_2(1)} \frac{H}{L}$	$\frac{f_1(1)}{f_2(1)} \frac{H}{L}$	$\frac{f_1(1)}{f_2(1)} \frac{H}{L}$	$\frac{f_1(1)}{f_2(1)} \frac{H}{L}$	$\frac{f_1(1)}{f_2(1)} \frac{H}{L}$
0.00	0.000	0.000	0.000	0.000	0.000	0.000
0.05	0.000	0.000	0.000	0.000	0.000	0.000
0.10	0.000	0.000	0.000	0.000	0.000	0.000
0.15	0.000	0.000	0.000	0.000	0.000	0.000
0.20	0.000	0.000	0.000	0.000	0.000	0.000
0.25	0.000	0.000	0.000	0.000	0.000	0.000
0.30	0.000	0.000	0.000	0.000	0.000	0.000
0.35	0.000	0.000	0.000	0.000	0.000	0.000
0.40	0.000	0.000	0.000	0.000	0.000	0.000
0.45	0.000	0.000	0.000	0.000	0.000	0.000
0.50	0.000	0.000	0.000	0.000	0.000	0.000
0.55	0.000	0.000	0.000	0.000	0.000	0.000
0.60	0.000	0.000	0.000	0.000	0.000	0.000
0.65	0.000	0.000	0.000	0.000	0.000	0.000
0.70	0.000	0.000	0.000	0.000	0.000	0.000
0.75	0.000	0.000	0.000	0.000	0.000	0.000
0.80	0.000	0.000	0.000	0.000	0.000	0.000
0.85	0.000	0.000	0.000	0.000	0.000	0.000
0.90	0.000	0.000	0.000	0.000	0.000	0.000
0.95	0.000	0.000	0.000	0.000	0.000	0.000
1.00	0.000	0.000	0.000	0.000	0.000	0.000

APPENDIX

ANALYTICAL EXPRESSIONS OF VARIOUS CASES

(a) For the case of a cylinder

$$\frac{1}{f_1(1)} \frac{f_2(1)}{f_1(1)} \frac{H}{L} = \frac{H}{L}$$

H_0 = Head Water Level

H_0 = Head of the liquid

H_0 = Head of the liquid

$$\frac{H}{L} = \frac{H}{L}$$

H = Head Water Level

L = Length

L = Length

3. Calculated values of K.

η	$(M_w/M_o)_M$	$(M_w/M_o)_A$	K
.2	.811	.933	.870
.25	.792	.910	.870
.30	.772	.886	.870
.35	.750	.860	.870
.40	.730	.833	.876

TABLE IX (a)

Values of K $L/D = 26.35$

η	$(M_w/M_o)_M$	$(M_w/M_o)_A$	K
.2	.772	.933	.829
.25	.754	.910	.829
.30	.737	.886	.830
.35	.718	.810	.834
.40	.700	.833	.840
.45	.683	.805	.847
.50	.665	.776	.856

TABLE IX (b)

Values of K $L/D = 19$

η	$(M_w/M_c)_M$	$(M_w/M_c)_A$	K
.2	.72	.933	.772
.25	.707	.910	.776
.30	.694	.886	.784
.35	.682	.860	.793
.40	.668	.833	.802
.45	.656	.805	.816
.50	.643	.776	.829

TABLE IX (c)

Values of K $L/D = 15$

η	$(M_w/M_c)_M$	$(M_w/M_c)_A$	K
.2	.677	.933	.726
.25	.667	.910	.734
.30	.657	.886	.741
.35	.648	.860	.754
.40	.639	.833	.767
.45	.629	.805	.781
.50	.619	.776	.796

TABLE IX (d)

Values of K $L/D = 14$

λ	$\lambda \left(\frac{1}{\sqrt{2}} \right)$	$\lambda \left(\frac{1}{\sqrt{2}} \right)$	λ
0.0	0.0	0.0	0.0
0.1	0.1	0.1	0.1
0.2	0.2	0.2	0.2
0.3	0.3	0.3	0.3
0.4	0.4	0.4	0.4
0.5	0.5	0.5	0.5
0.6	0.6	0.6	0.6
0.7	0.7	0.7	0.7
0.8	0.8	0.8	0.8
0.9	0.9	0.9	0.9
1.0	1.0	1.0	1.0

TABLE II

Values of λ for $\lambda = 1.0$

λ	$\lambda \left(\frac{1}{\sqrt{2}} \right)$	$\lambda \left(\frac{1}{\sqrt{2}} \right)$	λ
0.0	0.0	0.0	0.0
0.1	0.1	0.1	0.1
0.2	0.2	0.2	0.2
0.3	0.3	0.3	0.3
0.4	0.4	0.4	0.4
0.5	0.5	0.5	0.5
0.6	0.6	0.6	0.6
0.7	0.7	0.7	0.7
0.8	0.8	0.8	0.8
0.9	0.9	0.9	0.9
1.0	1.0	1.0	1.0

TABLE III

Values of λ for $\lambda = 1.0$

η	$(M_v/M_o)_M$	$(M_v/M_o)_A$	K
.2	.600	.933	.644
.25	.595	.910	.654
.30	.590	.886	.664
.35	.585	.860	.676
.40	.577	.833	.690
.45	.571	.805	.706
.50	.565	.776	.726

TABLE IX (e)

Values of K $L/D = 11$

η	$(M_v/M_o)_M$	$(M_v/M_o)_A$	K
.15	.800	.954	.839
.224	.782	.922	.849
.30	.753	.886	.849
.40	.712	.833	.855
.50	.674	.776	.869

TABLE IX (f)

Values of K $L/D = 21$

i	$\lambda_i(\sqrt{N})$	$\mu_i(\sqrt{N})$	ρ_i
100.	0.00.	0.00.	0.
105.	0.05.	0.05.	0.05.
110.	0.10.	0.10.	0.10.
115.	0.15.	0.15.	0.15.
120.	0.20.	0.20.	0.20.
125.	0.25.	0.25.	0.25.
130.	0.30.	0.30.	0.30.

TABLE II

$\lambda_i = \mu_i = \rho_i$ for $i = 1, 2, \dots, 100$

i	$\lambda_i(\sqrt{N})$	$\mu_i(\sqrt{N})$	ρ_i
100.	0.00.	0.00.	0.00.
105.	0.05.	0.05.	0.05.
110.	0.10.	0.10.	0.10.
115.	0.15.	0.15.	0.15.
120.	0.20.	0.20.	0.20.
125.	0.25.	0.25.	0.25.
130.	0.30.	0.30.	0.30.

TABLE III

$\lambda_i = \mu_i = \rho_i$ for $i = 1, 2, \dots, 100$

C. SAMPLE CALCULATIONS

1. Theoretical frequency of free-free bar vibrating in air.

$$f = \frac{m_n^2}{2\pi} \left[\frac{EK^2g}{\gamma} \right]^{1/2}$$

where m_n = 4.730 for first mode

E = Modulus of elasticity

= 580,000 psi for Lucite (approx.)

γ = Specific gravity = 1.18

g = 386 in/sec²

k = Radius of gyration of section

k^2 = .441

$$f = \frac{.0081}{6.28} \frac{580,000 \times .441}{11.05 \times 10^{-5}}^{1/2}$$

= 62.0 cycles per second

for 2 noded frequency of 52.7-inch cylinder.

2. Calculation of $\frac{M_w}{M_c}$.

$$\frac{M_w}{M_c} = \frac{M_T}{M_c} \left[\left(\frac{f_1}{f_2} \right)^2 - 1 \right]$$

where M_w = Added water mass

M_c = Displaced water mass

M_T = Mass of tube

For second mode of 52.7-inch bar

$$W_T = W_t/\text{in.} \times L + W_t.\text{clamp} + W_t.\text{Pickup} + W_t.\text{Exciting wire}$$

$$M_T = \frac{.497 \times 52.7 + .373 + .303}{16 \times 32.2}$$

$$= 0.0521$$

$$M_e = \frac{\pi D^2 L \gamma_w}{4g}$$

$$= \frac{3.14 \times 4 \times 52.7 \times 62.4}{4 \times 32.2 \times 1728}$$

$$= 0.186$$

$$f_1 = 170.5$$

$$f_2 = 86.2$$

$$\frac{M_T}{M_e} = .280(2.92)$$

$$= 0.820$$

3. Calculation of K.

By definition

$$K = \frac{M_w/M_e \text{ measured}}{M_w/M_e \text{ analytically computed}}$$

For 52.7" cylinder

when $\eta = .2$

$$\left(\frac{M_w}{M_e}\right)_M = .811$$

$$\left(\frac{M_w}{M_e}\right)_A = .933$$

Hence $K = \frac{.811}{.933} = 0.870$

For second mode of vibration

$$M_2 = M_1 \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right) = M_1 \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right)$$

$$M_2 = \frac{M_1 \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right)}{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots}$$

$$M_2 = 0.105 M_1$$

$$M_2 = \frac{M_1 \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right)}{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots}$$

$$M_2 = \frac{M_1 \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right)}{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots}$$

$$M_2 = 0.105 M_1$$

$$M_2 = 0.105 M_1$$

$$M_2 = 0.105 M_1$$

$$M_2 = 0.105 M_1$$

$$(M_2)_{max} = 0.105 M_1$$

$$M_2 = 0.105 M_1$$

3. Calculation of K

By definition

$$K = \frac{M_2}{M_1} = \frac{0.105 M_1}{M_1} = 0.105$$

For 2nd mode

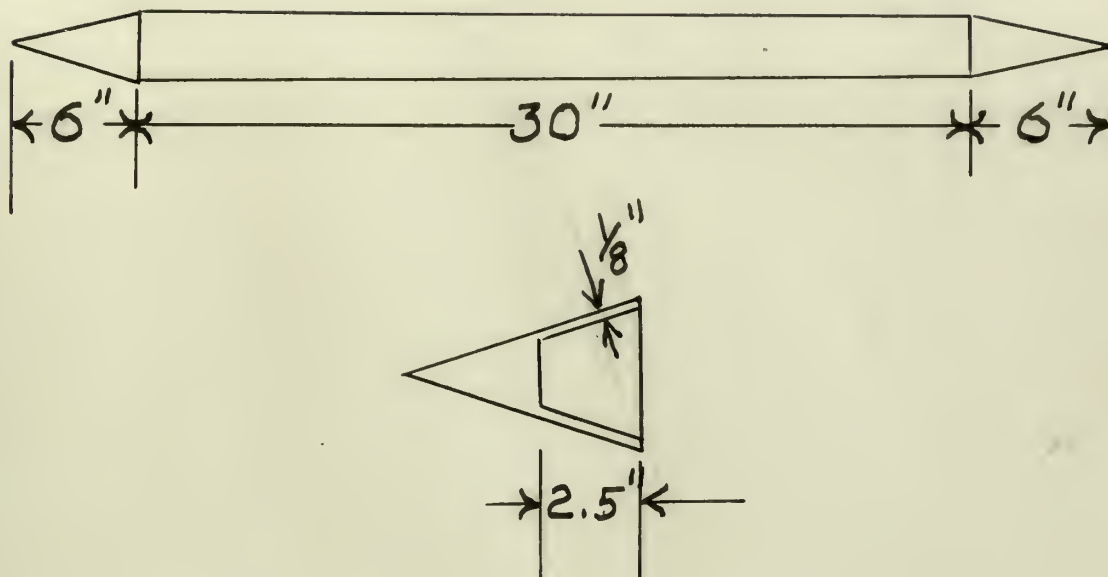
$$K = \frac{M_2}{M_1} = \frac{0.105 M_1}{M_1} = 0.105$$

$$K = \frac{M_2}{M_1} = \frac{0.105 M_1}{M_1} = 0.105$$

$$K = \frac{M_2}{M_1} = \frac{0.105 M_1}{M_1} = 0.105$$

$$K = \frac{M_2}{M_1} = \frac{0.105 M_1}{M_1} = 0.105$$

4.



The center of gravity of cones is approximately 3" from the base. Let us assume this body to be equivalent to a right circular cylinder 36 inches in length.

This assumption may be checked by calculating the air frequency of a 36-inch cylinder.

$$f = \frac{n^2}{2\pi} \left(\frac{EK^2g}{\gamma} \right)^{1/2}$$

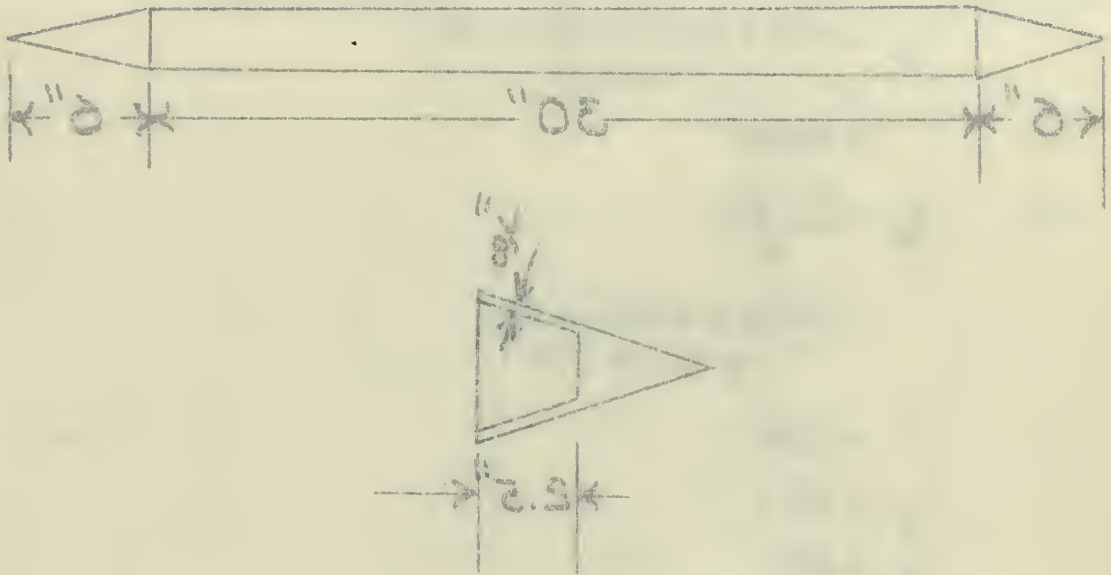
where $\frac{1}{2\pi} \left(\frac{EK^2g}{\gamma} \right)^{1/2} = 7,350$

$n = \frac{4,730}{L}$ for 2 noded frequency

$f = \left(\frac{4,730}{36} \right)^2 7350$

$f = 127 \text{ cps}$

Observed frequency was 131 cps, therefore equivalent length is 35.5 inches.



The center of gravity of a tapered shaft is located at a distance of 10 inches from the larger end. The center of gravity of a tapered shaft is located at a distance of 10 inches from the larger end. The center of gravity of a tapered shaft is located at a distance of 10 inches from the larger end.

$$x = \frac{L}{3} \left(\frac{D^2 + d^2 + Dd}{D^2 + Dd + d^2} \right)$$

$$x = \frac{L}{3} \left(\frac{D^2 + d^2 + Dd}{D^2 + Dd + d^2} \right)$$

$$x = \frac{L}{3} \left(\frac{D^2 + d^2 + Dd}{D^2 + Dd + d^2} \right)$$

$$x = \frac{L}{3} \left(\frac{D^2 + d^2 + Dd}{D^2 + Dd + d^2} \right)$$

$$x = \frac{L}{3} \left(\frac{D^2 + d^2 + Dd}{D^2 + Dd + d^2} \right)$$

The center of gravity of a tapered shaft is located at a distance of 10 inches from the larger end. The center of gravity of a tapered shaft is located at a distance of 10 inches from the larger end. The center of gravity of a tapered shaft is located at a distance of 10 inches from the larger end.

The water frequency now may be computed using Prof. Lewis' J, corrected by K for a 42-inch right circular cylinder

$$f = \left(\frac{4.730}{35.5} \right)^2 (7350) \left(\frac{M_T}{M_T + M_W} \right)^{1/2}$$

where $M_T = .036$

$$M_W = JKM_G \quad (12)$$

$$= (.947) (.839) (.120)$$

$$M_W = .0952$$

$$M_W + M_T = .1312$$

$$f = 68.9 \text{ cps.}$$

The experimental results gave a water frequency of 70 cps.

For the second mode

$$J = .923$$

$$K = .849$$

$$M_W = .0940$$

$$M_W + M_T = .1300$$

$$f = 187 \text{ cps}$$

Experimental results gave a water frequency of 188 cps.

The above expression may be simplified using the following relation:
for a linear elastic material:

$$\frac{1}{2} \left(\frac{d}{d\epsilon} \right) \left(\frac{d\sigma}{d\epsilon} \right) = \frac{1}{2} \left(\frac{d\sigma}{d\epsilon} \right) = \sigma$$

(2)

$$\sigma = \frac{1}{2} \left(\frac{d\sigma}{d\epsilon} \right)$$

$$\sigma = \frac{1}{2} \left(\frac{d\sigma}{d\epsilon} \right)$$

$$\sigma = \frac{1}{2} \left(\frac{d\sigma}{d\epsilon} \right)$$

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$$\sigma = \frac{1}{2} \left(\frac{d\sigma}{d\epsilon} \right)$$

$$\sigma = \frac{1}{2} \left(\frac{d\sigma}{d\epsilon} \right)$$

The experimental results give a value of $\sigma = 100$ MPa.

For the second case:

$$\sigma = \frac{1}{2} \left(\frac{d\sigma}{d\epsilon} \right)$$

$$\sigma = \frac{1}{2} \left(\frac{d\sigma}{d\epsilon} \right)$$

$$\sigma = \frac{1}{2} \left(\frac{d\sigma}{d\epsilon} \right)$$

$$\sigma = \frac{1}{2} \left(\frac{d\sigma}{d\epsilon} \right)$$

$$\sigma = \frac{1}{2} \left(\frac{d\sigma}{d\epsilon} \right)$$

Experimental results give a value of $\sigma = 100$ MPa.

D. ORIGINAL DATA

1. The following data was taken using the decade counter.

<u>MODE</u>	<u>FREQUENCIES</u>			<u>AVERAGE</u>
1	114	114	114	114
2	313	313	313	313
3	604	601	602	602
4	944	942	942	943
5	1352	1350	1351	1351

TABLE X

Air Frequencies 38-inch Cylinder

<u>MODE</u>	<u>FREQUENCIES</u>			<u>AVERAGE</u>
1	59	59	59	59
2	163	163	163	163
3	320	319	319	319
4	510	507	509	509
5	740	739	739.5	739.5

TABLE XI

Water Frequencies 38-inch Cylinder

TABLE I

1. The following data were taken at the various points.

DATE	TIME	TEMPERATURE	WIND	WAVE
1	11.4	11.1	11.4	11.4
2	11.3	11.3	11.3	11.3
3	11.2	11.2	11.2	11.2
4	11.1	11.1	11.1	11.1
5	11.0	11.0	11.0	11.0

TABLE II

2. The following data were taken at the various points.

DATE	TIME	TEMPERATURE	WIND	WAVE
1	11.4	11.1	11.4	11.4
2	11.3	11.3	11.3	11.3
3	11.2	11.2	11.2	11.2
4	11.1	11.1	11.1	11.1
5	11.0	11.0	11.0	11.0

TABLE III

3. The following data were taken at the various points.

MODE	FREQUENCIES				AVERAGE
1	182	183	183	183	183
2	496.5	495.5	496.5	496.5	496.5
3	933	936	935.5	936.5	936.0
4	1454.5	1453.5	1453	1454.5	1454.0

TABLE XII

Air Frequencies 30-inch Cylinder

MODE	FREQUENCIES				AVERAGE
1	98	97.5	97.5	97.5	97.5
2	268.5	269	267	268.5	268.5
3	510	513	513.5	511	512.0
4	810	809	808	807.5	808

TABLE XIII

Water Frequencies 30-inch Cylinder

DATE	DESCRIPTION	AMOUNT	DATE	DESCRIPTION	AMOUNT
1935	1935	1935	1935	1935	1935
1935	1935	1935	1935	1935	1935
1935	1935	1935	1935	1935	1935
1935	1935	1935	1935	1935	1935

TABLE III

Table showing the results of the experiment.

DATE	DESCRIPTION	AMOUNT	DATE	DESCRIPTION	AMOUNT
1935	1935	1935	1935	1935	1935
1935	1935	1935	1935	1935	1935
1935	1935	1935	1935	1935	1935
1935	1935	1935	1935	1935	1935

TABLE IV

Table showing the results of the experiment.

MODE	FREQUENCIES		AVERAGE
1	205	205	205
2	553	552.5	553
3	1036	1035.5	1036

TABLE XIV

Air Frequencies 28-inch Cylinder

MODE	FREQUENCIES		AVERAGE
1	112	112	112
2	305	305	305
3	581.5	581.5	581.5
4	916	917	916.5

TABLE XV

Water Frequencies 28-inch Cylinder

	DATA	DATE	TIME	LOC
1	200	200	200	1
2	200	200	200	2
3	200	200	200	3

VII. DATA

DATA FROM THE FIELD

	DATA	DATE	TIME	LOC
1	200	200	200	1
2	200	200	200	2
3	200	200	200	3
4	200	200	200	4

VIII. DATA

DATA FROM THE FIELD

MODE	FREQUENCIES			AVERAGE
1	298	299.5	298.5	299
2	800	801	800	800
3	1457	1458	1457	1457

TABLE XVI

Air Frequencies 22-inch Cylinder

MODE	FREQUENCIES			AVERAGE
1	171	171	172	171
2	465	466	465	465
3	854	855	855	855

TABLE XVII

Water Frequencies 22-inch Cylinder

DATE	PERIOD			NO.
1952	1952	1952	1952	1
1953	1953	1953	1953	2
1954	1954	1954	1954	3

TABLE III

Water Properties of the Samples

DATE	PERIOD			NO.
1952	1952	1952	1952	1
1953	1953	1953	1953	2
1954	1954	1954	1954	3

TABLE IV

Water Properties of the Samples

MODE	FREQUENCIES				AVERAGE
1	104	105	105	104	104.5
2	291	290	291	290	290.5
3	561.5	562	562	562	562
4	891	894	893	893	893
5	1278	1276	1276	1277	1277

TABLE XVIII

Air Frequencies 40-inch Cylinder

MODE	FREQUENCIES				AVERAGE
1	54.0	54.0	53.5	54.0	54.0
2	150.0	151.0	151	151	151
3	296	294	295	295	295
4	477	476.5	476	476	476
5	693	691	692	691.5	692

TABLE XIX

Water Frequencies 40-inch Cylinder

ITEM	PERCENTAGE				ADJUSTED
1	104	104	104	104	104.0
2	104	104	104	104	104.0
3	104.2	104	104	104	104.2
4	104	104	104	104	104.0
5	104	104	104	104	104.0

TABLE III

The Programmed Air-Sea System

ITEM	PERCENTAGE				ADJUSTED
1	104.0	104.0	104.0	104.0	104.0
2	104.0	104.0	104.0	104.0	104.0
3	104	104	104	104	104.0
4	104.2	104	104	104	104.2
5	104	104	104	104.2	104.2

TABLE IV

Water Programmed Air-Sea System

MODE	FREQUENCIES			AVERAGE
1	131	131	131	131
2	353	351	352	352
3	659	658	658	658
4	1016	1020	1014	1017
5	1417	1419	1415	1417

TABLE XX

Air Frequencies 30-inch Cylinder
with 6-inch Conical Ends

MODE	FREQUENCIES			AVERAGE
1	70	70	70	70
2	188	188	188	188
3	358	355	352	355
4	555	555	555	555
5	796	790	790	792

TABLE XXI

Water Frequencies 30-inch Cylinder
with 6-inch Conical Ends

DATE	DESCRIPTION	AMOUNT
1911	1911	100
1912	1912	100
1913	1913	100
1914	1914	100
1915	1915	100

TABLE II

Water Treatment Plant, Chicago
 1911-1915

DATE	DESCRIPTION	AMOUNT
1911	1911	100
1912	1912	100
1913	1913	100
1914	1914	100
1915	1915	100

TABLE III

Water Treatment Plant, Chicago
 1911-1915

2. The following data was taken using the high speed movie camera.

MODE	FREQUENCIES		AVERAGE
1	61.7	61.7	61.7
2	170.5	170.5	170.5
3	332.0	332.0	332.0
4	539.5	539.5	539.5
5	787.5	787.5	787.5

TABLE XXII

Air Frequencies 52.7-inch Cylinder

MODE	FREQUENCIES		AVERAGE
1			
2	86.2	86.2	86.2
3	169.1	169.1	169.1
4	279.0	279.0	279.0
5	419.0	419.0	419.0

TABLE XXIII

Water Frequencies 52.7-inch Cylinder

3. The following table shows the results of the tests made on the

TEMPERATURE	RELATIVE HUMIDITY	PERCENTAGE	WATER
100.0	100.0	100.0	1
100.1	100.1	100.1	2
100.2	100.2	100.2	3
100.3	100.3	100.3	4
100.4	100.4	100.4	5

TABLE 1

Results of tests made on the

TEMPERATURE	RELATIVE HUMIDITY	PERCENTAGE	WATER
100.0	100.0	100.0	1
100.1	100.1	100.1	2
100.2	100.2	100.2	3
100.3	100.3	100.3	4
100.4	100.4	100.4	5

TABLE 2

Results of tests made on the

FIGURE VI
PHOTOGRAPHS OF TYPICAL TWO NODED FREQUENCIES



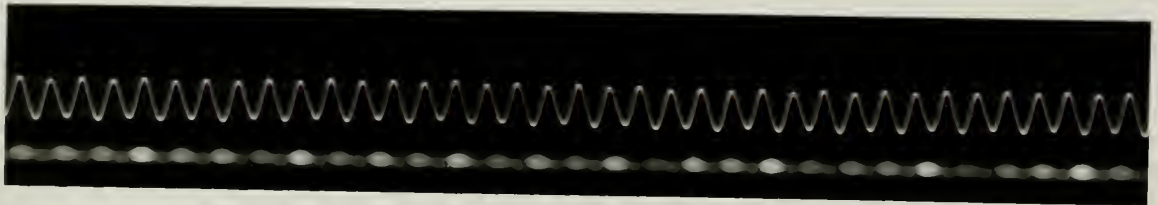
TWO NODED WATER FREQUENCY



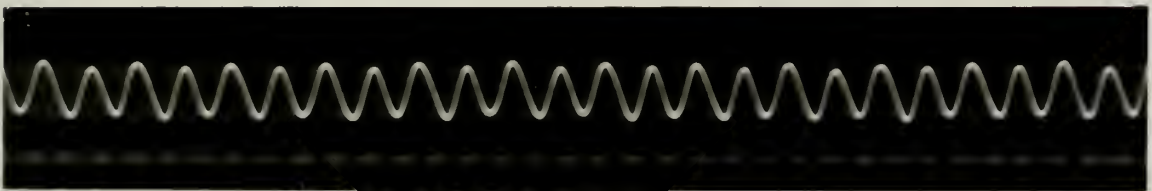
TWO NODED AIR FREQUENCY

FIGURE VII

PHOTOGRAPHS OF TYPICAL SIX NODED FREQUENCIES



SIX NODED AIR FREQUENCY



SIX NODED WATER FREQUENCY

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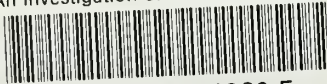
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